

II.4-MUSKROUT MUSKINGUM ROUTING

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Introduction

Muskingum routing is a storage routing method based on the storage equation which is an expression of continuity:

$$I - O = \frac{dS}{dt} \tag{1}$$

where I is the inflow rate
O is the outflow rate
S is storage
t is time

The expression for storage in a reach of a stream used in the Muskingum method is:

$$S = K[xI + (1-x)O] \tag{2}$$

where K and x represent storage parameters

To apply Muskingum routing to a reach it is first necessary to determine values for the parameters K and x which are used to describe the storage characteristics of the reach. Values of K and x are normally derived analytically from historical flood records. The parameter K is a storage constant expressing the ratio between storage and discharge and is usually expressed in hours. It may also be viewed as the lag or travel time through the reach. The dimensionless parameter x is indicative of the relative importance of inflow and outflow to storage. Like K it is also referred to as a storage constant. Although the Muskingum method is not as inherently flexible as the Lag and K method of routing it is adequate for many reaches and

routing situations.

Additional information on the developments of Muskingum routing and the derivation of K and x values for a reach can be found in Linsley, Kohler and Paulhus (1949), (1975), Lawler (1964) and Gray (1973).

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Muskingum Routing Algorithm

Muskingum routing is based on the general expression of storage in Equation 2. Using subscripts 1 and 2 to indicate the beginning and end of routing interval t_r the storage equation may be expressed as:

$$\frac{I_1 + I_2}{2} - \frac{O_1 + O_2}{2} = \frac{S_2 - S_1}{t_r} = \frac{K[xI_2 + (1-x)O_2] - K[xI_1 + (1-x)O_1]}{t_r} \quad (3)$$

I_1 , I_2 and O_1 are known quantities at the beginning of a routing operation while O_2 is the unknown and sought after routed discharge. In a more simplified form the Muskingum routing equation usually appears as:

$$O_2 = c_0 I_2 + c_1 I_1 + c_2 O_1 \quad (4)$$

where

$$c_0 = - \left[\frac{Kx - .5t_r}{K - Kx + .5t_r} \right] \quad (5)$$

$$c_1 = \frac{Kx + .5t_r}{K - Kx + .5t_r} \quad (6)$$

$$c_2 = \frac{K - Kx - .5t_r}{K - Kx + .5t_r} \quad (7)$$

The coefficients c_0 , c_1 and c_2 are computed from K, x and t_r input values. The sum of these coefficients will always equal 1.0.

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Limits on Parameter Values

According to Lawler (1964) the routing time interval for which Muskingum routing works best is:

$$2Kx \leq t_r \leq K \quad (8)$$

Implied from this relationship is:

If this restriction is not met then there are no possible values of t_r that will satisfy the conditions of Equation 8. The Muskingum routing Operation checks the two conditions but will perform the routing even if these conditions are not met.

During execution negative outflows can result if conditions of Equations 8 and 9 are not met. If negative outflows occur then the routing time interval is divided into four equal time subintervals. Using linear interpolation the values of inflow at the intermediate times are calculated. After calculating new c_0 , c_1 and c_2 coefficients based on the smaller time interval Muskingum routing is done on these four subintervals to calculate the outflow at the end of the regular routing interval. If this outflow is positive then normal execution continues. If it is still negative then the outflow at the end of the time interval is recalculated by linear interpolation using the two previous outflows. If this is the first time step for a given run then the outflow at the end of the time interval is set equal to the outflow at the beginning of the time interval. If the outflow is still negative then it is set equal to zero and normal execution continues.

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Comment on the Use of Muskingum Routing

A basic assumption of Muskingum routing is that K remains constant at all flows. This assumption is useful as well as convenient and Muskingum routing with a constant K generally produces adequate routed flows for most reaches. However serious routing errors will occur in situations where this assumption does not hold true. Muskingum routing should not be applied to reaches where the storage-flow relationship, the basis for assuming that K is constant, is nonlinear. Other methods of routing such as Lag and K , Layered Coefficient or Dynamic Routing (Fread, 1978) should be considered for reaches that exhibit distinctly nonlinear storage-flow relationships.

Cunge (1969) and Buil (1967) developed variations to the Muskingum method that allow K and x to be a function of discharge, Q , thus the storage-flow relationship can be non-linear. These papers also discuss how the Muskingum coefficients can be estimated based on physical characteristics of the channel.

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